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A LOW COST WIDEBAND
ANALOG MULTIPLIER

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WALTER ALEN HUTCHENS

Submitted in partial fulfillment of
the requirements for the degrees of
Master of Science in Electrical Engineering
and
Master of Science in Naval Architecture

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WALTER ALEN HUTCHENS

Submitted to the Department of Electrical Engineering and the Department of Naval Architecture on 25 May 1967 in partial fulfillment of the requirements for the degrees of Master of Science in Electrical Engineering and Master of Science in Naval Architecture.

ABSTRACT

A highly accurate low cost analog multiplier using cadmium sulfide photoconductors has been designed and tested. Use of a bridge configuration for the photoconductors affords cancellation of several types of errors and permits bandwidth to be extended since the photoconductors respond to an increasing light level more rapidly than to a decreasing light level. In addition, four quadrant multiplication with single input polarity is provided.

A static accuracy of 8 millivolts and a zero product error of 2 millivolts were achieved. The frequency for 1% dynamic error was 1 Hz. and the error at 10 Hz. was under 5%. Deficiencies in the test version leading to frequent lamp burn-out and reduced bandwidth are discussed and corrective measures for these problems are presented; The redesigned multiplier should have a 1% error bandwidth of 7 Hz. or more without the problem of lamp burn-out.

The semiconductor physical principles behind photoconductor operation are discussed with special attention being given to the effects detrimental to multiplier use.

Thesis Supervisor: George C. Newton
Title: Professor of Electrical Engineering

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INTRODUCTION

Multiplication of two variables is the most difficult of the basic functions to implement satisfactorily in analog computation. An ideal analog multiplier would have high static and dynamic accuracy, wide bandwidth and yet be low in cost and simple; no device presently meets these conflicting requirements[1].

The aim of this thesis was to design and construct an analog multiplier having:

1) a static accuracy over the full range of both variables of 0.1% or better,
2) a 1% error bandwidth of 50 Hz. or more and,
3) a cost-per-product of $50.00 or less.

This goal was not fully achieved for reasons which will be discussed later; however, the approach described is sound, and it is believed that the shortcomings of this version could be eliminated in a subsequent effort.

THEORY

An analog multiplier is a device which realizes the relation

$$Z = kXY \tag{1}$$

where X and Y are variables and k is a constant determined by system and device parameters. In an electronic analog system the most convenient value for k is usually that value for which Z reaches full scale when both X and Y are at full scale, although this is not always the case[2].

The realization of (1) in terms of voltages requires a device for which the voltage gain is a precise linear function of a controlling voltage. Since no such device exists in elemental form, it is necessary to linearize a non-linear device. Suppose a device, E, has a transfer function

$$g = f(x)$$

where f(x) is understood to be a monotonically increasing function. Then if we insert the device in a feedback loop which provides x and give the device an input Y_o, as shown in Figure One,

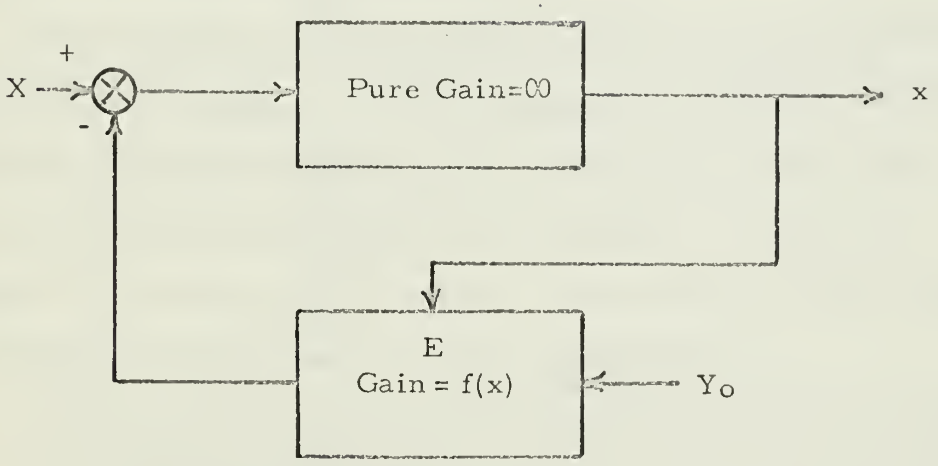


Figure One GENERAL FEEDBACK MULTIPLIER

we have available a "predistorted" x as an implicit function of the input variable X [3].

This elemental block is the basis of a great many accurate multipliers. It may be employed in two ways: (a) The input to the non-linear element, E, is switched among Y_o ... Y_n while its output is sampled to obtain kXY_o ... kXY_n; this requires that the gain f(x) be held constant over the sampling cycle and that some sort of smoothing be employed at the product outputs. Or (b) additional elements, E_1 ... E_n, having the transfer relationship f(x) receive the predistorted common input x but different inputs, Y_1 ... Y_n; This requires a number of identical elements but permits simple circuitry. Such a multiplier is analogous to the well known servo-multiplier.

The multiplier to be described here uses the second approach. The element employed is a bridge arrangement of Cadmium Sulfide, (CdS) photoconductors driven by two groups of miniature lamps. A committed operational amplifier is employed to restore the output to full scale, ($\pm$ 10 volts). The composite transfer characteristic of this element is shown in Figure Two.

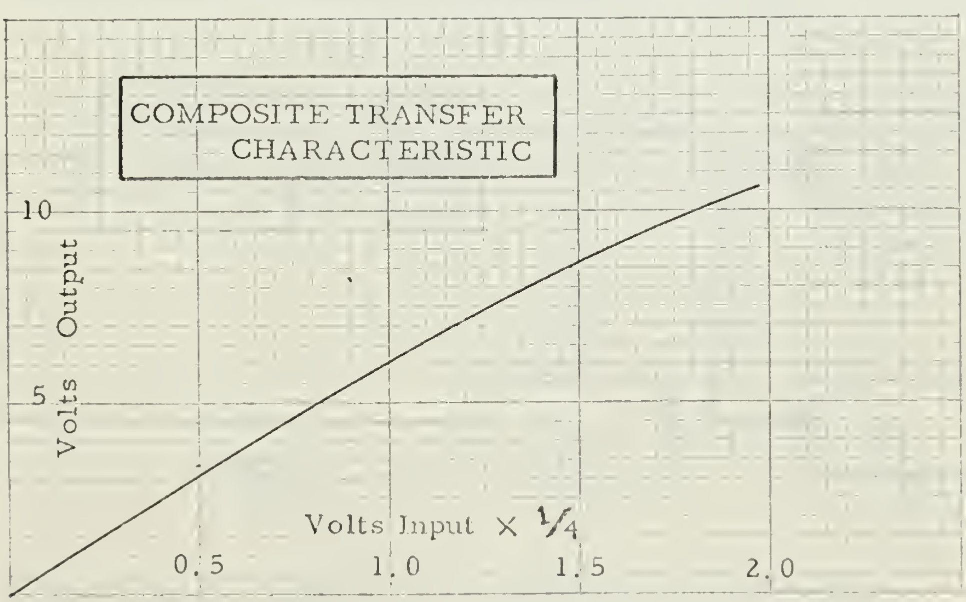


Figure Two

Figure Three shows a simplified circuit diagram of this element while Figure Four is a block diagram of the complete multiplier employing it.

Several other approaches to the realization of a photoconductive multiplier are possible; Figure Five shows a simplified schematic of a multiplier resulting from a previous project in the Electronic Systems Laboratory[4].

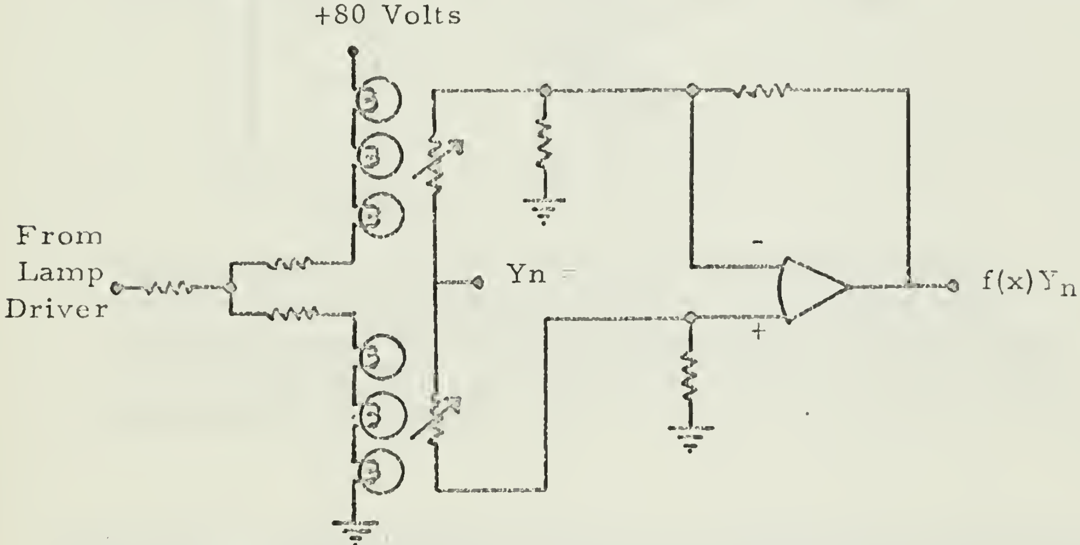


Figure Three BASIC MULTIPLIER ELEMENT

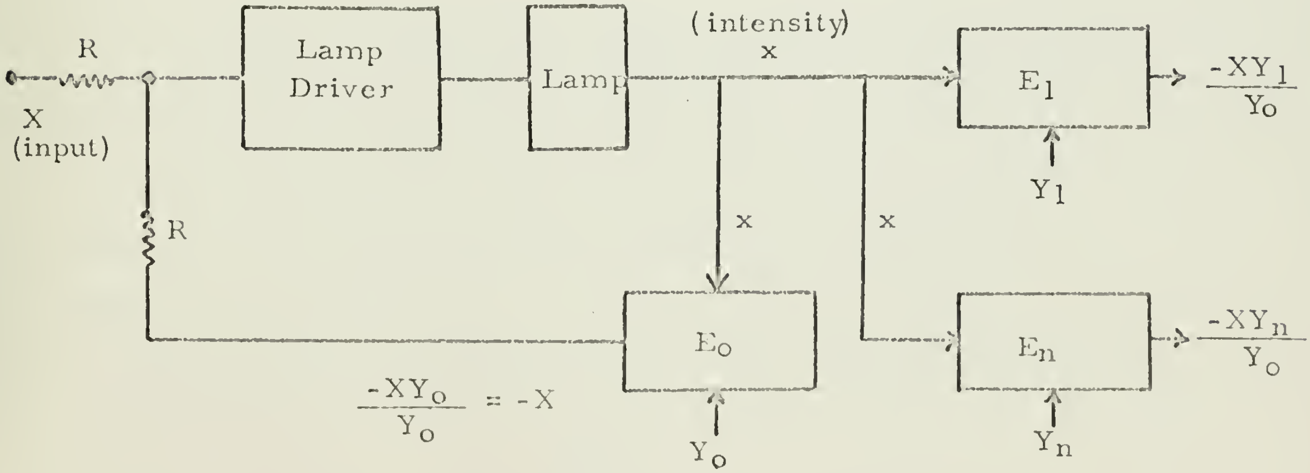
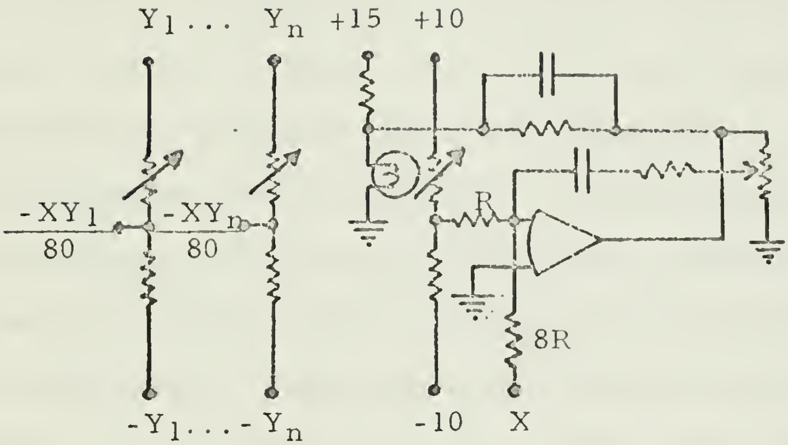


Figure Four COMPLETE FEEDBACK MULTIPLIER



The reason for the scale factor of 8 in this design is the need to restrict the range of voltages across each photoconductor in order to reduce voltage effect. The cells used are CL 605L - 020 and the lamp is a #338 as in the design presented in this thesis.

Figure Five SIMPLIFIED SCHEMATIC OF ESL FOUR QUADRANT MULTIPLIER

PHOTOCONDUCTOR SELECTION

Stated loosely, a photoconductor is a device having a conductance directly proportional to the incident light intensity. Unfortunately, there are several reasons that this is not exactly true; these must be taken into account in the selection and application of photoconductors to an accurate analog multiplier[5].

Photoconductors exhibit substantial temperature coefficients. For selected CdS cells of the most stable type, typical values are in the range of -0.1% to -0.5% per degree centigrade at 28° C. at moderate light levels. Cadmium selenide, (CdSe) cells are substantially worse; -1% per degree is the proper order of magnitude[6].

A "light history" effect is important. After a period of darkness, some time is required for the cell to reach its final resistance. For the cells used in this multiplier the change is approximately +2% in 48 hours following a one hour exposure to light sufficient to give a resistance of 40,000 ohms. This effect is less serious at lower resistances; here too CdSe cells are worse.

Cell time constants are strongly dependent on the light level and are substantially shorter for increasing light intensity than for decreasing intensity. In one test, the ratio of "rising" time constant to "falling" time constant was 1:10 between the same resistance limits.

Finally, there is a significant "voltage effect". This means

that the cell resistance is affected by the applied voltage; this effect is of the order of -0.1% per volt for the cell used in this multiplier and is substantially worse for cells having a narrower interelectrode gap and again for CdSe.

Since stability and accuracy were prime considerations, a cell type previously shown to give superior performance in these characteristics was selected. The Cl 605L - 020 manufactured by the Clairex Corporation is particularly suited to this application because (1) it uses a very stable cadmium sulfide photoconductive material and (2) it has an unusually wide interelectrode gap, which minimizes voltage effect. Cells of this type have been used in previous multipliers designed by the Electronic Systems Laboratory so that a standard was available by which to judge the merits of the circuit.

CIRCUIT DESIGN

Temperature coefficient, light history, and voltage effect suggest an arrangement in which the errors caused by one cell can be canceled by those of another. For the temperature coefficient and light history this is relatively simple to accomplish since it is simple to mount the cells so that they will all be at the same temperature and all of the cells of a particular multiplier will have the same light history. Voltage effect cancellation is somewhat more difficult to obtain because this requires that the opposing cells have the same applied voltage but even this can be done.

The great difference between the time required for rise of conduction and the time for it to fall favors a differential circuit in which one cell could be rising for output changes of either direction.

These constraints specify a balanced bridge whose output is the current differential between two cells operating at the same applied voltage but different light levels. Figure Six shows a simplified schematic of the multiplier which resulted from this approach. Operation is quite simple: with no X input the lamps are equal in brightness; an input produces a difference in lamp brightness which results in an amplifier output X' cancelling the X input current(assuming infinite lamp driver gain). Each product amplifier receives a differential input current proportional to the difference of cell resistances and the product input, and this current differential is canceled by a proportional output voltage through a feedback resistor.

Provided that the steady-state range of cell resistance is small, voltage effect is canceled in each bridge; the same is true of light history effects. Temperature coefficients do not so cancel because they affect the difference of cell resistance in each bridge; however, they cancel between the master bridge and each product bridge since the resistance changes are equal. (Since temperature coefficients exhibit a substantial scatter, cancellation is not perfect[7].)

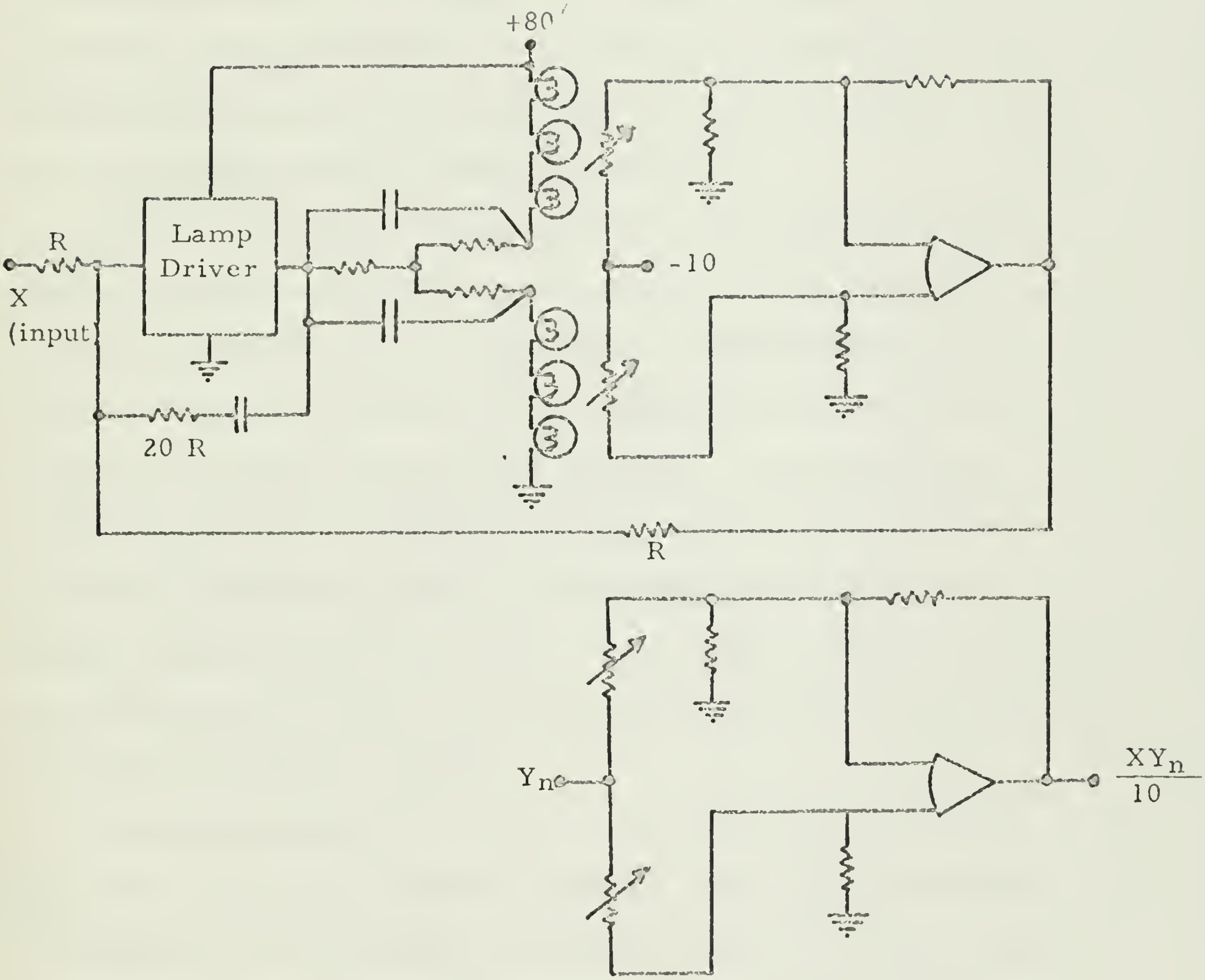


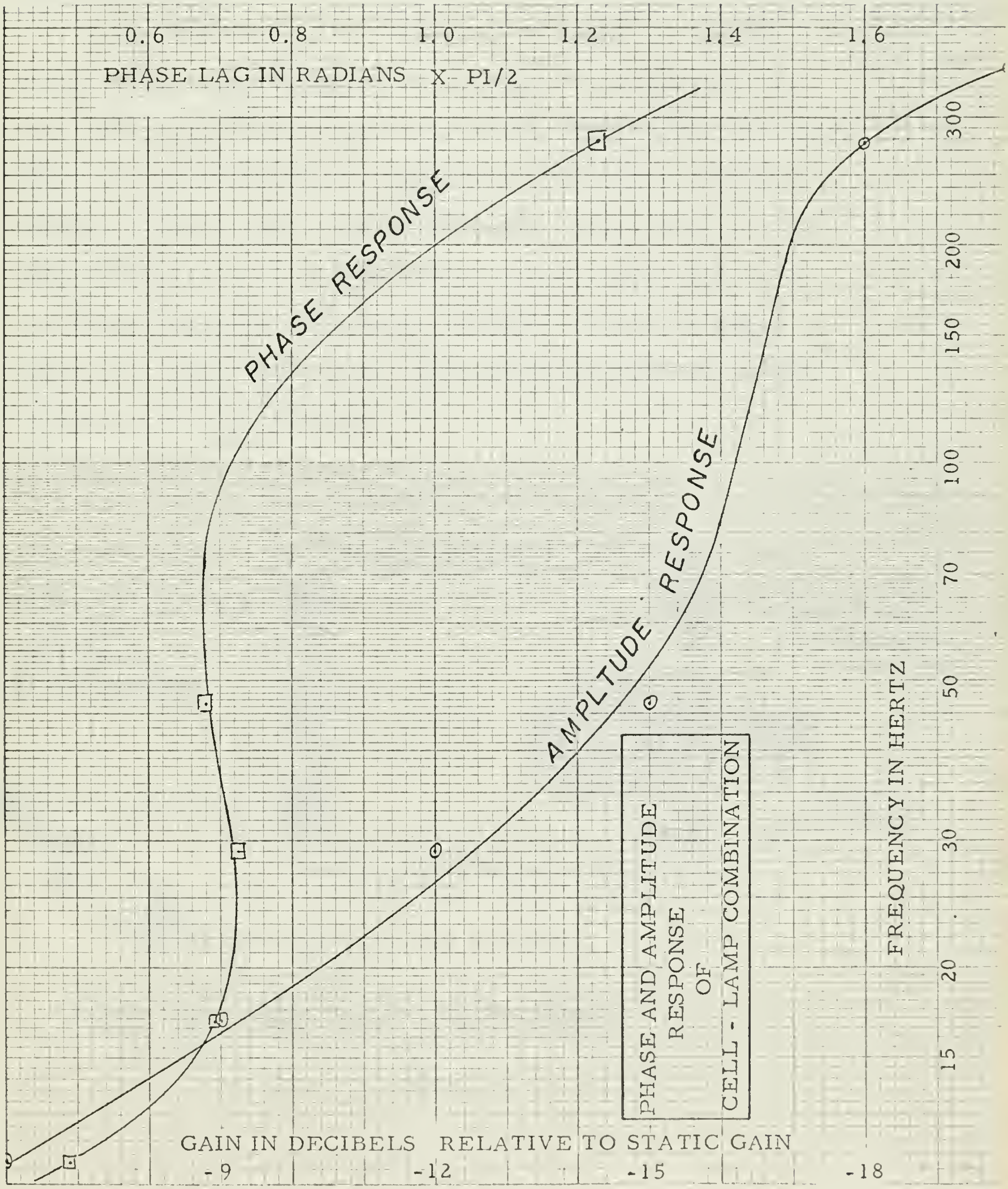
Figure Six SIMPLIFIED SCHEMATIC OF BRIDGE-TYPE FOUR QUADRANT MULTIPLIER

Provided that substantial over-ranging is possible, i.e., that the lamp driver and lamps are capable of exceeding the range of intensities necessary for the full static range, any change of output required is met by a decrease of resistance of one cell or the other. This arrangement gives the maximum speed of which the cells are capable; see Appendix One for design calculations.

Several light sources were considered. Neon lamps are extremely fast; however, they exhibit several types of instability which makes them unsuitable for this application. Electroluminescent strips and panels are ineffective when operated at frequencies high enough to be outside the multiplier bandwidth. Electroluminescent diodes with sufficient output for multiplier use are not presently available (several were tested); however, they have excellent frequency response and are quite stable. It seems likely that outputs and efficiencies will rise with further development, and this would make them the method of choice[8].

Incandescent lamps were finally chosen; their only serious disadvantage is their poor frequency response. Since they can be made to <u>increase</u> output as rapidly as desired by applying sufficient filament voltage, this is less of a disadvantage than it might otherwise be.[9] As previously explained, a rapid <u>increase</u> of brightness is required to obtain the best performance from the bridge cell configuration while a rapid <u>decrease</u> is less critical. Figure Seven shows the amplitude and phase characteristics of the lamp driver - lamp -

cell combination.



MECHANICAL DESIGN

In the test version, all components are physically located on a pair of DEC plug-in modules; this is more than enough space and with smaller operational amplifiers one card would probably be sufficient. The lamps and cells are mounted in two machined aluminum cavities salvaged from an earlier multiplier design; they were modified to mount holders for three #338 miniature lamps rather than the single lamp used in the earlier version. As shown in Figure Eight, each cell is mounted in a brass cartridge adjustable axially by means of an 8-56 leadscrew.

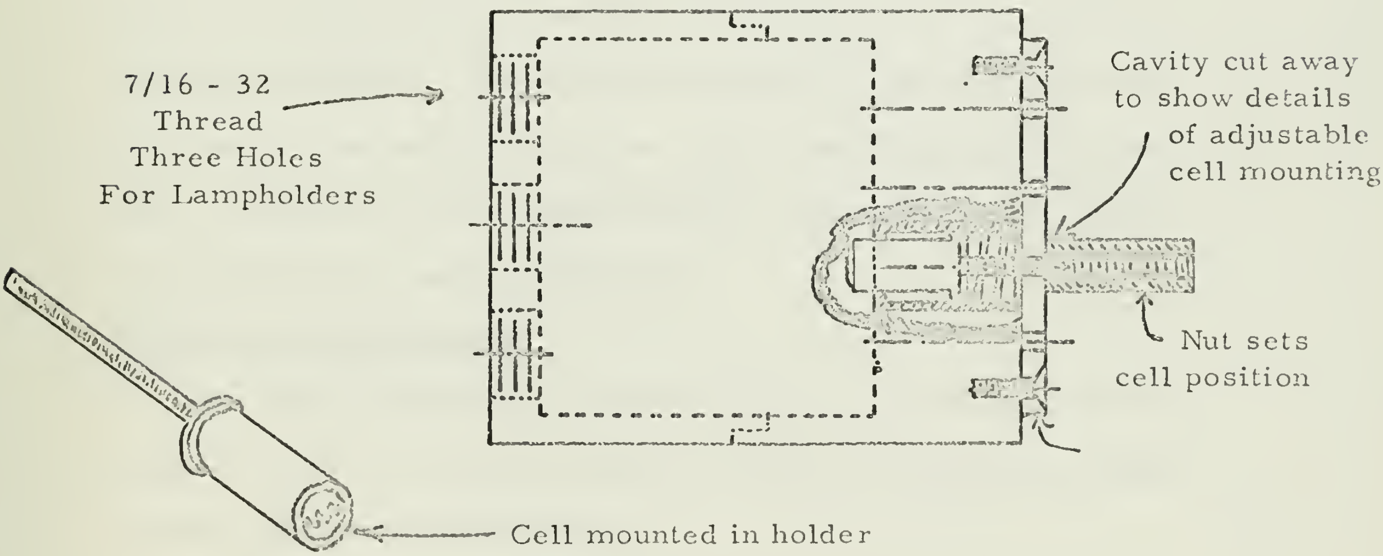


Figure Eight CELL HOLDER AND CAVITY

A spring surrounding the leadscrew takes up backlash, and the leads from the cell pass within the spring and out through the slot in which the leadscrew operates.

As might have been anticipated, the cell leads are subject to abrasion and frequent short circuit; when this happens in the master bridge it causes failure of one or more lamps.

PERFORMANCE

Two products were wired and the multiplier was aligned as described in Appendix Two. Results of performance tests are compared below with a previous Electronic Systems Laboratory four-quadrant multiplier design[10].

As can be seen, frequency response is not satisfactory. Another serious deficiency is the frequency with which lamps are burned out by input transients. The only other problem is the short circuits mentioned in the preceding section. Since lamp replacement necessitates realignment (which requires about $\frac{1}{2}$ hour to perform), it must be eliminated in a useful multiplier.

CORRECTION OF DEFICIENCIES

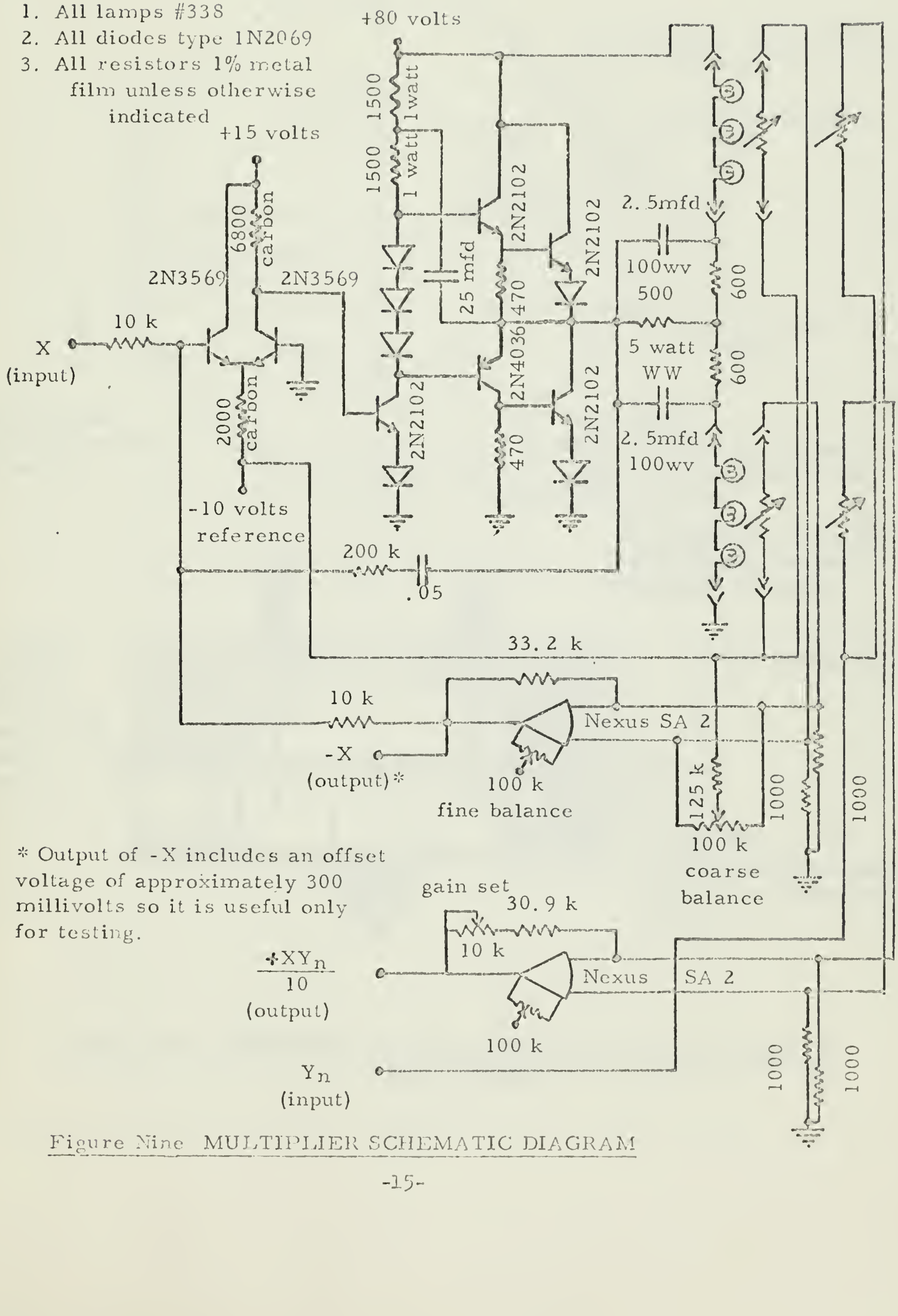
The three difficulties discussed above (poor frequency response, frequent short circuits, and lamp burn out due to transients) appear to have straightforward remedies.

The poor frequency response is evidently purely the result of insufficient lamp driver gain coupled with incorrect driver compensation. Use of a high-gain amplifier with proper compensation should

TABLE ONE

	Bridge Multiplier	ESL Multiplier
Static error, worst case over entire range of both products (Immediate)	± 8 millivolts (peak error at 1 Hz.)	40 millivolts (static)
(24 hours after 1 hour warmup	± 40 millivolts	Not known
Zero error; either product at zero, other at full scale - worst case (Immediate)	less than 2 millivolts	30 millivolts
(24 hours after 1 hour warmup)	less than 10 millivolts	30 millivolts
Frequency for 1% dynamic error (Common multiplier) Full scale input	1 Hz.	2 Hz.
Performance at 10 Hz.	less than 5% dynamic error	Maximum frequency for full scale output
Temperature Coefficient	Not known but believed small.	2 millivolts /°F. or less

1. All lamps #338
2. All diodes type 1N2069
3. All resistors 1% metal film unless otherwise indicated



* Output of -X includes an offset voltage of approximately 300 millivolts so it is useful only for testing.

Figure Nine MULTIPLIER SCHEMATIC DIAGRAM

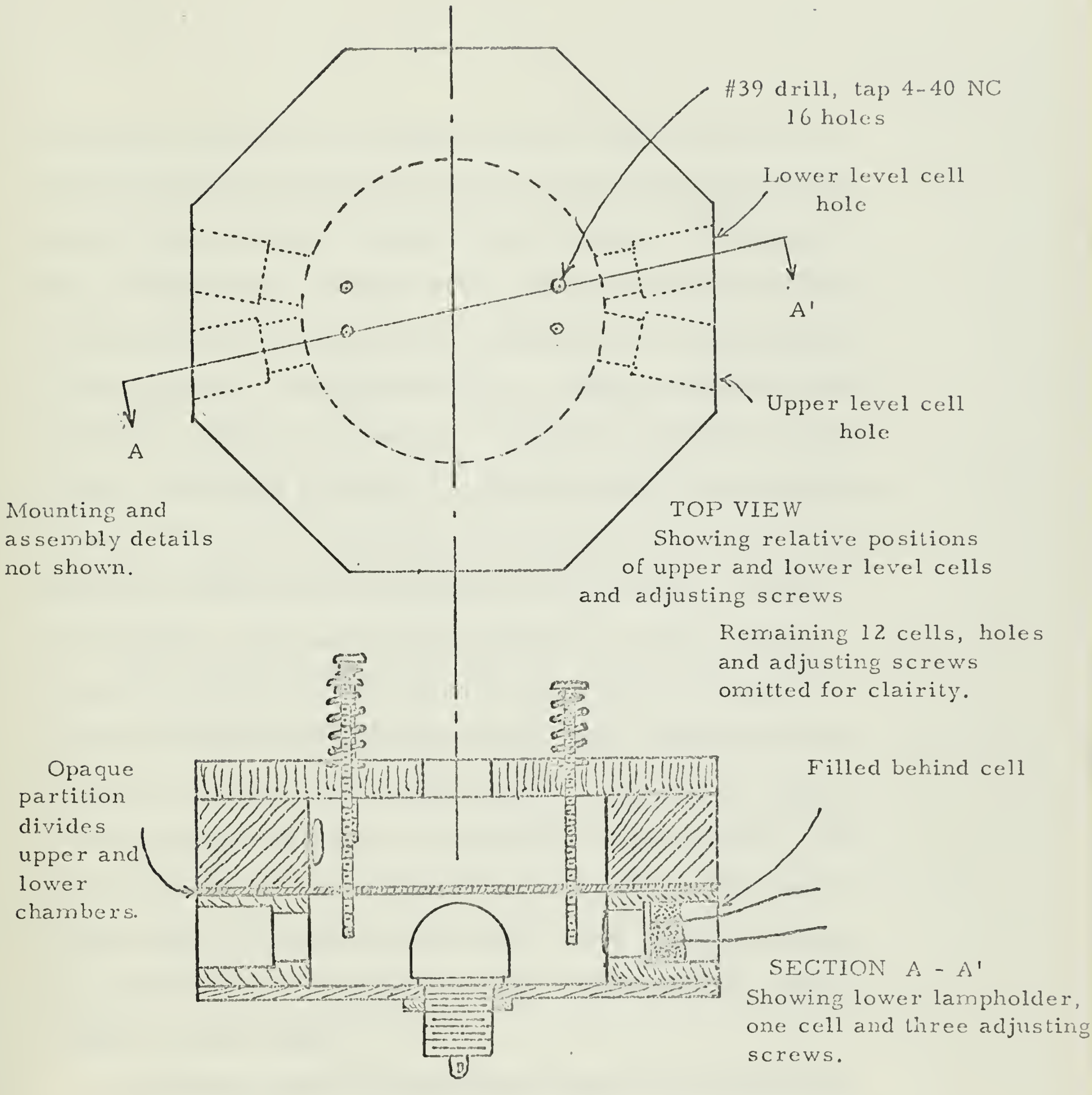


Figure Ten SUGGESTED CAVITY DESIGN FOR SEVEN PRODUCT MULTIPLIER

raise the frequency for 1% dynamic error to 7 Hz. or more. See Appendix Three for detailed calculations. The short-circuit problem is more difficult. The most obvious answer is to insulate the cell leads with a material having greater abrasion resistance; the present insulation is teflon tubing which is a poor performer in this respect. But movable cells are a damned nuisance and make the cavity expensive to produce. A suggested alternative cavity design, which would eliminate the abrasion problem and be simple to construct either as a prototype on the usual machine tools or in moderately large quantities justifying other methods, is shown in Figure Ten. The lamp-to-cell light path is radial, and the adjusting screws partially occlude the cell face, giving precisely the same effect as movable cells. Some adjustment of dimensions, screw size, and location might be necessary in order to obtain the proper combination of adjustment range and rate. The cells should be mounted by filling the cell back with an opaque epoxy compound to exclude ambient light. This configuration has the added advantage of giving about the closest possible thermal coupling between cells[11].

Because the cells are substantially closer to the lamp than in the previous cavity designs, only one lamp is required to obtain the recommended range of resistances[12].

The final problem is the lamp burnout by input transients; it seems probable that this can be controlled by reducing the maximum

lamp excitation voltage. The present value, about 25 volts, was determined experimentally before the static lamp temperature was known and it is clearly too high for actual operating conditions. The correct value is that which will raise a lamp from the minimum static temperature to the maximum allowed slewing temperature when applied through the chosen capacitor[13]. A good start would be to reduce the +80 volt supply to about +50 volts and adjust the lamp series resistors to give the correct zero product temperature and slewing range. New capacitors should then be chosen to give the best cell response to a square wave of about 5 Hz. without reducing lamp life.

What is needed is a compensation providing "square" response of lamp intensity to increasing input voltages (minimum rise time with little or no overshoot). The proper capacitor choice would provide an approximation to this condition but a more sophisticated approach would be direct feedback of lamp intensity to the driver input. This could be accomplished in at least two ways, either of which would be at the expense of more complex circuitry:

1) A photodiode or phototransistor could be exposed to the light in each cavity; these have very much faster response than the highly stable photoconductors used in the main feedback loop[14]. Their lack of linearity and stability would be much less important if they were applied to controlling the intensity of the lamp since

1) cont.

it would affect only the slewing speed of the multiplier. A block diagram of this system is shown in Figure Eleven (a).

2) The lamp intensity could be obtained indirectly by sensing the filament resistance. One way of accomplishing this would be to excite the lamp with a small current at a relatively high frequency, say 50 kHz. and detect the resulting voltage. This could be fed back to the driver input as with the photodiode; the added complexity would be somewhat greater than with that method. This system is block diagramed in Figure Eleven (b).

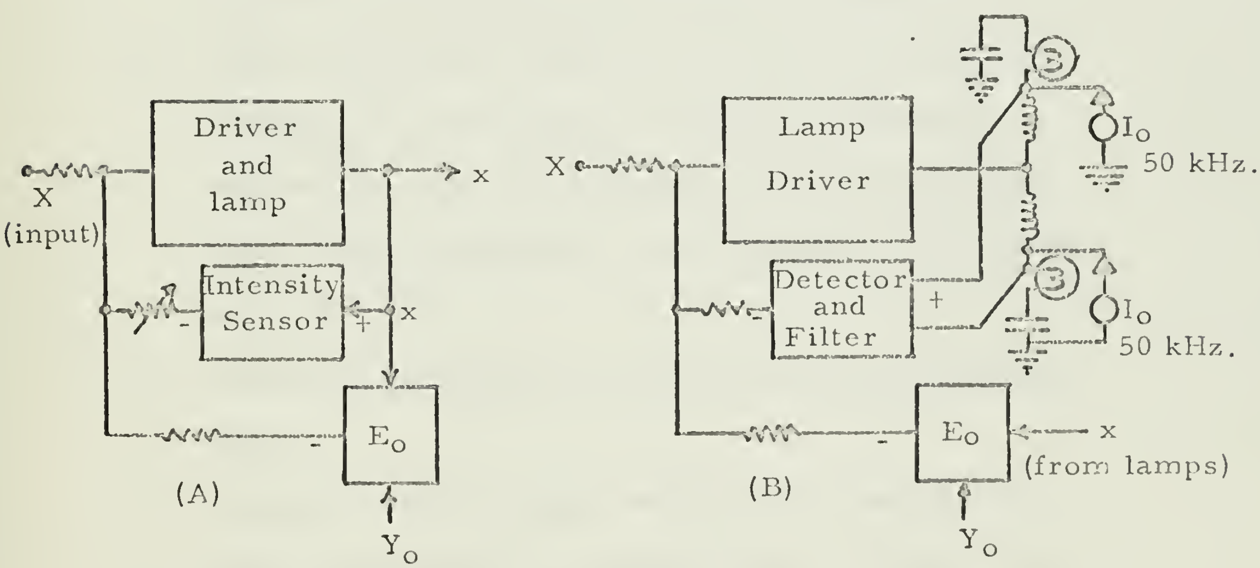


Figure Eleven FEEDBACK OF LAMP INTENSITY

CONCLUSIONS AND RECOMMENDATIONS

The multiplier design described herein is a straightforward low cost approach to a device with a static error of 0.1% or less and a 1% dynamic error frequency of 5 to 10 Hz. With further investigation more sophisticated compensation or lamp control strategies would probably extend the bandwidth. The mechanical design of the light cavities needs to be changed both to eliminate short circuits and to simplify construction. Specific recommendations are:

1) Replace the present lamp driver circuit with an operational amplifier having a gain of 70 db. or more, followed by a simplified lamp driver having only the voltage gain required to expand the voltage range of the operational amplifier to the range chosen to drive the lamps.

2) Sutdy in systematic fashion the effect of various combinations of peak voltage and coupling capacitance on combined cell and lamp frequency response and on lamp life. Better combinations than that used in this design can be found[15].

3) Examine systematically the various compensation alternatives.

4) Determine whether a light cavity can be designed with fixed cells trimmed by "shadow screws". If this is not

4) cont.

possible, errors resulting from electronic trimming by parallel and/or series resistance should be determined and a cavity using fixed non-adjustable cells should be used.

5) Examine the possibility of using a less stable photoconductive material (since this configuration cancels most of the errors from this source) having faster response characteristics[16]. If a fixed-cell package is used, the small diameter of the Clairex type 600 package becomes less important and a material not available in that package might well be chosen. This would be particularly true for a radial-path design such as that described earlier. The CL 705HL cell (TO-5 package) should be investigated since it appears to be about 5 times faster than the present cell but has only twice the temperature coefficient. Voltage effect would have to be studied,however.

APPENDIX ONE

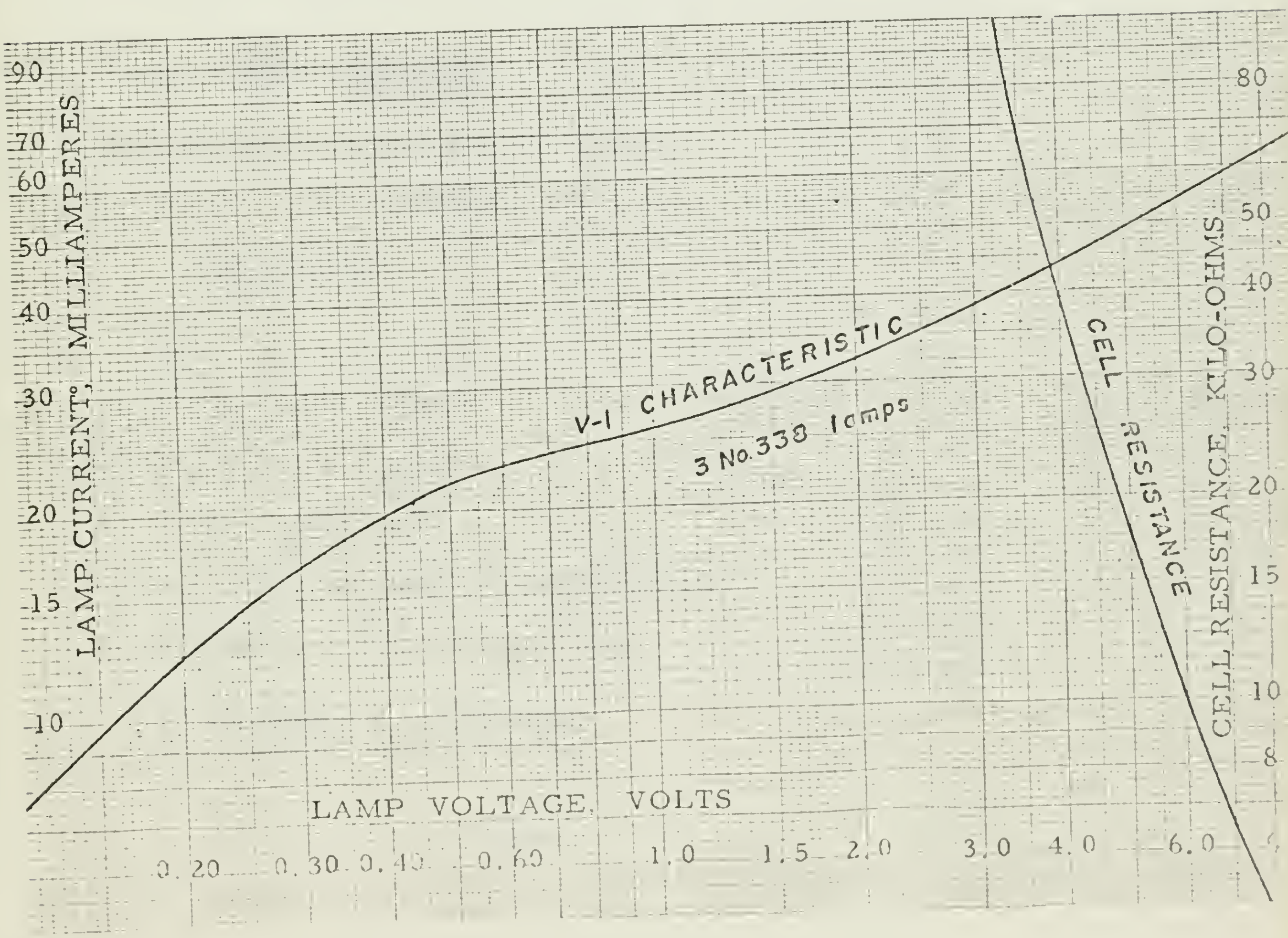
MASTER LOOP LAMP VOLTAGE DIVIDER CALCULATIONS

Initially the following constraints were to met:

1) For full scale static deflection the cell resistance were to be 10 k and 22.5 k respectively.

2) Maximum voltage on the lamps during slewing will be 10 volts; this is to be accomplished with the maximum available input to the lamp divider, 40 volts.

From constraint (1) above and the cavity V-I and transfer characteristic below, we see that the lamp voltages are 5.98 volts



for 10 k cell resistance and 4.66 volts at 22.5 k. Further, we find the lamp currents at these voltages are 54 ma. at 5.98 volts and 48 ma. at 4.66 volts.

Thus from the diagram at the right:

1) $I = 54 - 48 = 0.006$ amperes

2) $\dfrac{80 - V - 5.98}{R_a} = 0.054$ amperes

3) $\dfrac{V - 4.66}{R_a} = 0.048$ amperes

adding we get:

4) $\dfrac{69.36}{R_a} = 0.102$ amperes

5) $R_a = \dfrac{69.36}{.102} = \underline{\underline{680 \text{ ohms}}}$

Applying the second constraint we get:

6) $V = 80 - 10 - 0.071(680) = 70 - 48.3$

$= \underline{\underline{21.7}}$ volts

and

7) $I + I_1 = 0.071$ amperes

8) $IR_b = V = 21.7$ volts

9) $21.7 - I_1(680) = V$

10) I_1V_1 is on the lamp V-I characteristic.

By trial and error it is found that:

$\underline{I_1 = 0.0296}$ $\underline{V_1 = 1.6 \text{ volts}}$

Then from (7),

$I = 0.071 - -.0296 = \underline{0.0414}$ amperes

Substituting in (8):

$IR_b = 0.0414\ R_b = 21.7.$ Rearranging

$$R_b = \frac{21.7}{0.0414} = \underline{523 \text{ ohms}}$$

APPENDIX II

ALIGNMENT AND TESTING

As constructed, there are 4 useful adjustments on the master bridge and 4 on each product. Alignment requires some care but the procedure is straightforward; Figure Nine is a detailed schematic of the master control loop, and one product showing all adjustments. Alignment is carried out as follows:

1) Set all cell screws and all bias trimmers at midrange. Set product gain pots about 5 turns from the maximum gain position, and apply all voltages. Connect $Y_1 \ldots Y_n$ to -10 volt reference voltage and ground the X input.

2) Do not touch the master cell screws. Attempt to use the master bias trimmers (coarse and fine), which will affect all products equally, and the individual product bias trimmers to bring all products to zero without touching the product cell screws. Where necessary, turn product cell screws in opposite directions equally to make this possible.

3) Connect a .1 Hz., 20 volt peak - to - peak sine wave to the X input. Connect a precision voltage divider as shown in Figure Twelve (a). The scope display will be the product error divided by two; the millivoltmeter is needed to accurately set the zero.

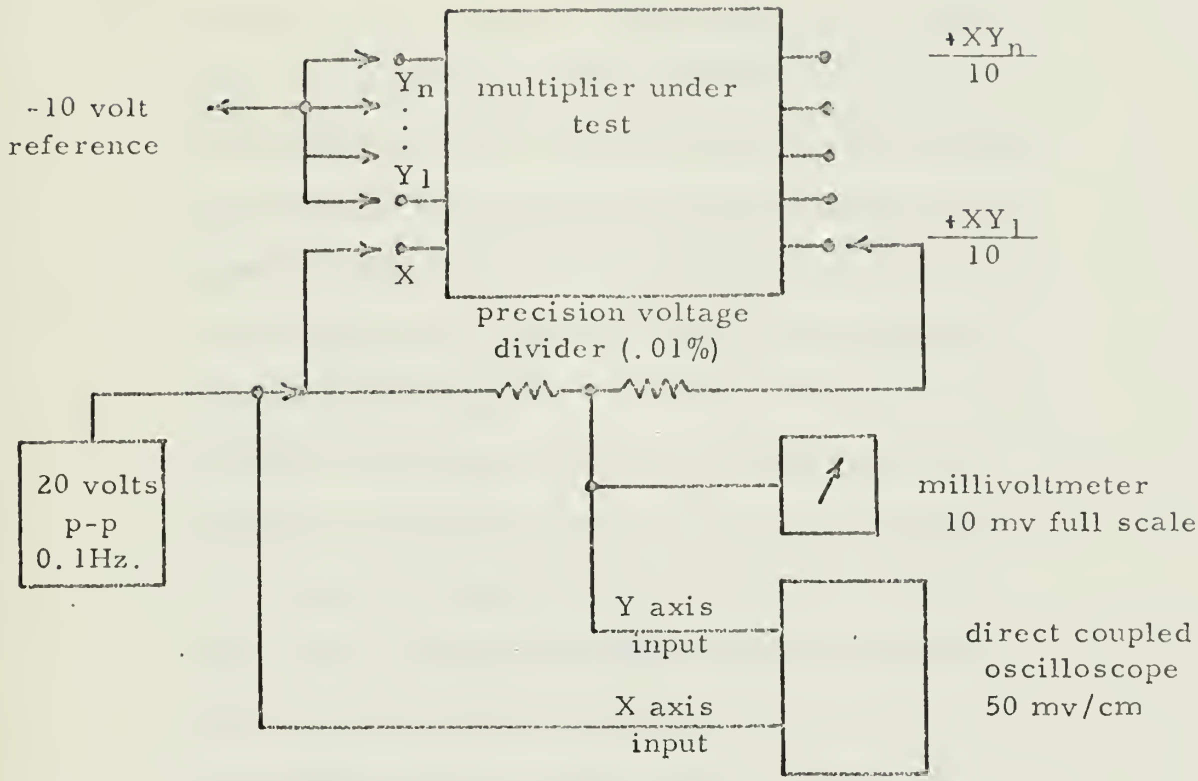


Figure Eleven (a) TEST CONNECTIONS FOR COMMON PRODUCT

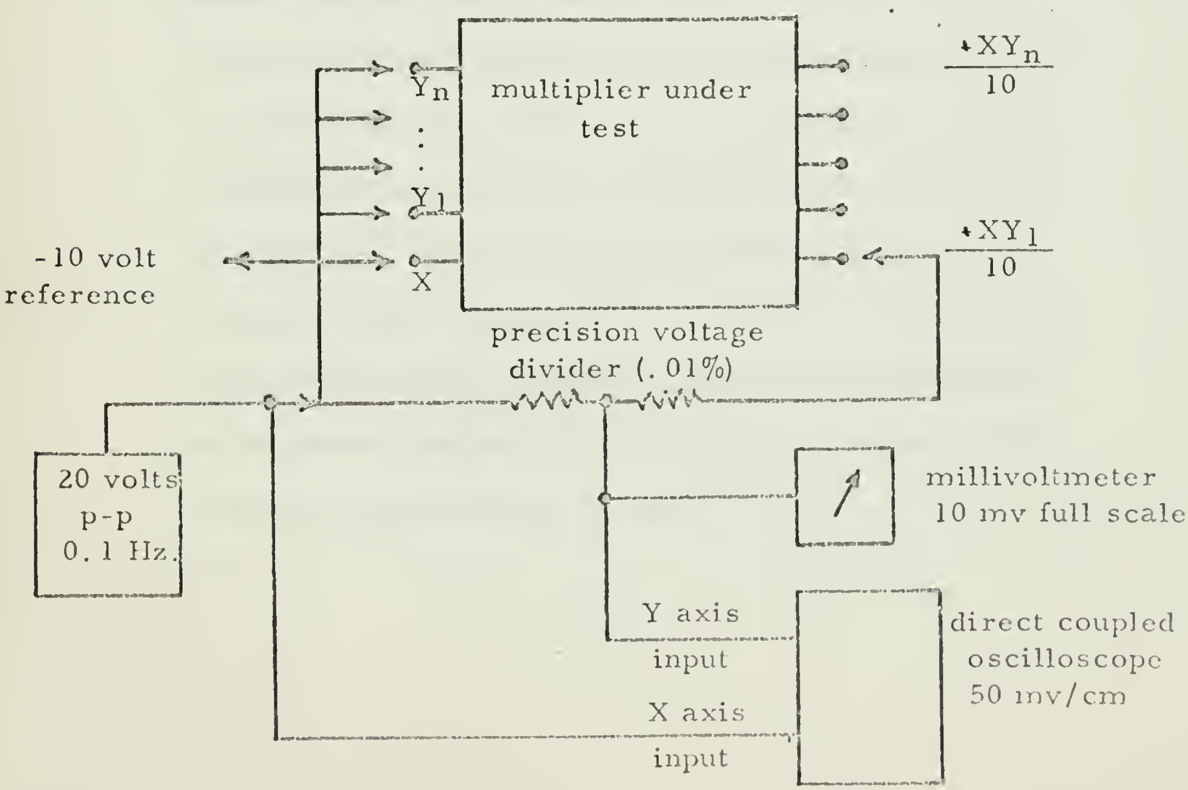


Figure Eleven (b) TEST CONNECTIONS FOR SLAVE PRODUCTS

4) Go through and look at all the product errors on the scope. You should see nearly straight lines passing through zero at center scale and having various slopes. On one product at a time use the gain trimmer and the amplifier offset trimmer alternately to bring the ends of the trace to the same error with essentially zero error at center scale.

5) At this point the error traces will probably have a predominant curvature; either most of the traces will be bent up at the ends of most of them will be bent down. If so, pick out an average trace and use the master zero set trimmer to move it 100 millivolts in the direction that its center should be moved for a flat trace. Use one master cell screw to bring the center of the trace back 50 millivolts and the other to finish the job. The trace will now probably have a slope; use the product gain trimmer to eliminate this, and finish by re-zeroing using the product offset trimmer.

The error trace will now be flatter than it was; repeat the procedure until satisfied. Since these adjustments on the master loop affect all products, the mean curvature will now be reduced to zero.

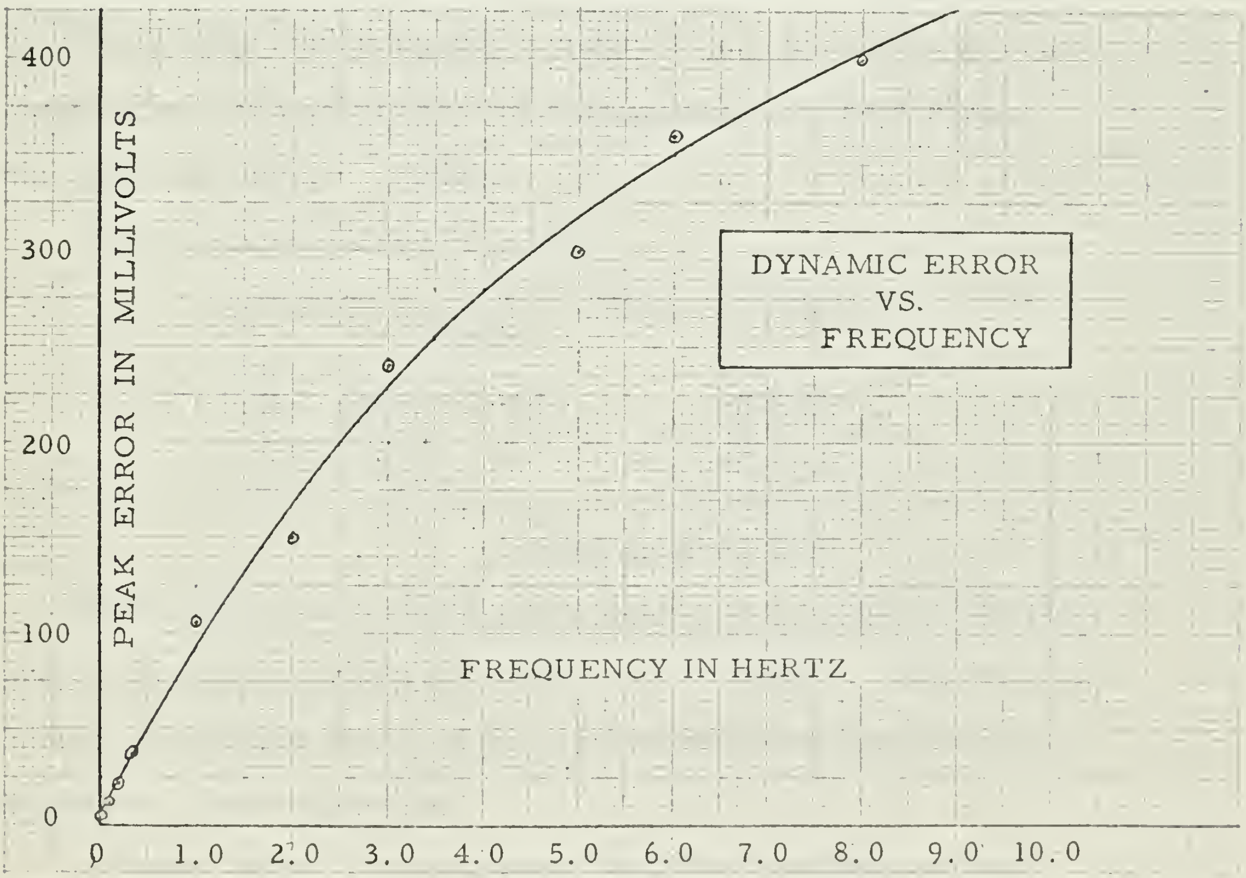
6) Look again at the other products and retrim them for flattness (gain) and zero error (offset). If any of them have a substantial curvature, use the offset screw to move the trace in the direction opposite to that required to bring its center into line, and use the product cell screws to bring it back. Retrim the product for flatness as before.

7) All product errors should now be less than 10 millivolts peak. If any product cannot be brought down to this error, a serious cell mismatch is indicated; this will usually result in an excessive curvature on one product which cannct be eliminated. Before changing cells, try changing product operational amplifiers since the amplifier offset current may vary enough to bring an otherwise impossible curvature into line.

8) Now interchange the common and slave inputs, connecting the 0.1 Hz. signal to all of the slaves, $Y_1 \ldots Y_n$, and the X input to the -10 volt reference voltage. Reconnect the voltage divider as shown in Figure Twelve (b) and look at each of the product errors. Probably they will all be less than 10 millivolts; those which are not have a cell with excessive voltage effect and which will have to be changed.

The foregoing looks quite complicated but in practice it is simple to do. Provided that the lamp burn-out problems previously described are eliminated, alignment should seldom be necessary since considerable compensation for change in temperature is built into the design.

The testing arrangement described above was used throughout the development of the multiplier to determine both static and dynamic errors. As aligned, the multiplier constructed had the dynamic error versis frequency characteristic shown in Figure Thirteen.



APPENDIX III

STABILITY AND COMPENSATION CALCULATIONS

Figure Seven shows the phase and amplitude response of the lamp-cell combination as initially constructed. For this test the driver amplifier was given a fixed voltage gain of 4. The signal amplitude was 2% of the full scale. The test was made by measuring the increase of input signal to maintain a constant output as frequency increased Measurements were terminated when lamp burn-out occurred at 395 Hz.

The compensation used with the driver amplifier gave it a gain asymptotic to 20 (i.e., 26 db.) above 16 Hz. and to a rise of 6 db. per octave below that frequency. Since the loop was stable for this compensation, it appears that the loop gain reaches 0 db. before the phase lag reaches π radians.

The loop gains above 16 Hz. are:

Operational Amplifier gain (flat)	+24.3 db.
D.C. Lamp-cell gain	-8.6 db.
Driver gain	+26 db.
Total gain:	+41.7 db.

So the lamp-cell combined gain relative to D.C.must be -41.7 db. or less at the point where the phase lag is π radians. This is not obvious from Figure Seven and the test clearly should have been extended to higher frequencies.

A better choice of compensation (with the knowledge of loop stability at 26 db. of driver gain) would be a characteristic

asymptotic to 20 db. (to allow for phase lag added by the driver by changing the compensation) above 100 Hz. and rising at 6 db per octave below that frequency.

At 100 Hz. the net gain would be:

Operational amplifier gain	+24.3 db.
D.C. Lamp-cell gain	-8.6 db.
Driver gain	+20.0 db. (roughly)
Lamp-cell gain relative to D.C. (Figure Seven)	-16.2 db.
Total Loop Gain:	+19.5 db.

Since 1% dynamic (voltage) error corresponds to 40 db. of net loop gain, the 1% error frequency would be:

$$\frac{40 - 19.5}{6} = 3.42 \text{ octaves below } 100 \text{ Hz.}$$

$$F_{1\%} = \frac{100}{2^{3.42}} = \frac{100}{13.35} = 7.5 \text{ Hz.}$$

From Figure Seven it appears that the added phase lag would be well tolerated below 100 Hz.

Now consider a steeper characteristic, for example, 6 db. per actave below 100 Hz. and asymptotic to 12 db. per octave below 50 Hz. For calculation purposes, we can guess at the frequency for 1% dynamic error by assuming a break at 75 Hz. and a characteristic asymptotic to 12 db. per octave below that.

We need 20.5 db. of added gain for a net of 40 db., hence we must go:

$$\frac{26.5}{12} = 1.7 \text{ octaves below } 75 \text{ Hz.}$$

$$F_1\% = \frac{75}{2^{1.7}} = \frac{75}{3.25} = \underline{\underline{23.1 \text{ Hz}}}.$$

Such compensation would be well worth trying, though the added phase lag would probably be unacceptable.

APPENDIX FOUR

SEMICONDUCTOR PHYSICS OF PHOTORESISTOR OPERATION

In this appendix, the effects observed in photoresistor operation will be explained in terms of basic physical principles; this writer was not able to find such a detailed explanation when doing the actual work[17].

A photoconductor is a two terminal element having a resistance which varies inversely with the incident light level. Present day commercial photoconductors take the form of a film of semiconductor material deposited on an insulating substrate and enclosed in a sealed envelope which is usually evacuated. All-glass envelopes are most common,but modified TO-8, TO-5, and TO-18 metal-and-glass envelopes are also available. Connections to the semiconductor film are made by means of two metal electrodes (often interdigitated) plated onto the film; this produces an ohmic (non-rectifying)contact. Wire leads are soldered to these electrodes and brought out through the envelope.

Not surprisingly, the most important factor affecting cell performance is the semiconductor material chosen. At present, the most common materials are cadmium sulfide (CdS) and cadmium selenide (CdSe);.These materials operate in the same fashion, however each has its particular advantages and disadvantages. The following discussion principles applies to both materials.

By definition, a semiconductor material has available a few (but not many) electrons and "holes" to serve as carriers of an electric current. Under these conditions, the allowed energy levels in a perfect, pure crystal are only those represented by bound or "valence" electrons and free or "conduction" electrons. Semiconductor materials used in photoresistors are actually composed of highly imperfect polycrystaline films. Additionally, they contain deliberately introduced impurities (doping agents), which introduce imperfections at the atomic level by causing local distortions of the crystal lattice.

Both types of imperfections introduce additional allowed energy levels between the valence band and the conduction band. Two general types of such levels occur.

Donor and acceptor levels result from a doping agent having either an extra or a deficient electron. Clearly since a donor atom has an electron not required to form bonds with adjacent electrons this electron is very nearly free. This implies an allowed energy very near the conduction band, a "donor level". The complementary argument holds for acceptor levels which lie near the valence band.

The other possibility, trapping and recombination levels, is the result of crystal imperfections as mentioned above. An electron may enter such a level from either the valence band or the conduction band. If it enters from the conduction band, it gives up part of

its energy tolattice vibrations,(heat); thereafter either of two processes may occur: (1) The electron may recombine with a hole from the valence band giving up the remainder of its energy or, (2) the electron may be thermally reexcited to the conduction band. If (1) predominates the level is a "recombination level" while if (2) is more probable, it is a "trapping level". With this greatly simplified picture of photoresistor operation we can proceed to discuss some of the effects observed in such elements.

Let us begin by examining the basic phenomenon of photoresistivity. Suppose we have a photoresistor which has been stored at absolute zero and in total darkness,and further suppose that all trapping and recombination levels were empty. Now, we raise the temperature of the photoresistor to room temperature. Electrons are thermally excited from the valence band and from the donor levels to the higher levels (trapping or recombination). Some electrons will gain enough energy to enter the conduction band; this is the reason that even in total darkness a cell never has infinite resistance.

If the cell is now illuminated with light of the proper wavelength many electrons are excited directly from the valence band and the nearby acceptor levels into the conduction energy band. Two important effects related to the growth of conduction will be discussed; one is the short-term effect of photoconductor "time constant" and the other is the "light history effect" which is observed

when a photoconductor is illuminated for periods of seconds to days.

As previously stated, both trapping and recombination energy levels may be thought of as resulting from physical defects in the crystal lattice; they are thus associated with specific sites in the photosensitive material. Further, these sites may be thought of as having a specific "capture cross section". Thus, the rate at which electrons are captured at trapping or recombination centers is a monotonically increasing function of the density of electrons in the conduction energy band; i.e., of the conductivity. Growth of conductivity thus represents an excess of electrons entering the condition band over those being captured and this growth will continue until an equilibrium is reached between the rate at which electrons are excited and that at which they are captured. If the capture probability were a constant, this growth would be along an exponential curve; the real situation isn't so simple. Equilibrium with those trapping and recombination centers near the conduction band is reached fairly rapidly; but with those centers having energy levels much below the conduction band, equilibrium takes a longer time. It is for this reason that the typical photoconductor conductivity growth curve shows a variable "time constant" which increases with illumination time.

We have so far assumed that only electrons need consideration and that all capture sites are either trapping or recombination centers independent of time. Actually, neither assumption is

correct. A capture site is a "recombination" site if it contains a hole because a captured electron will then give up its energy and will not re-enter the conduction band. Since the conduction process actually includes a few holes as well as the many electrons, as conduction continues, some sites will acquire a hole and be converted from trapping levels (from which an electron could be thermally re-excited to the conduction band) into recombination centers. It is easy to see that this process results in a slight decrease in the rate of thermal excitation of electrons and hence a slight decrease in the total conductivity. At high illumination levels, thermal re-excitation is less significant so that this "light history effect" becomes less serious.

Since short-term changes in illumination cause relatively little change in the population of the intermediate energy levels, this is a long-term process.

Because of the light history effect, analog multipliers employing photoconductors typically require several days with application of all supply voltages to fully stabilize.

Another important effect which must be considered in the application of photoconductors to accurate analog multiplier devices is the "voltage effect". For various reasons as outlined above, substantial numbers of electrons exist in energy levels just below the conduction band in such an element under illumination. If a voltage is now applied producing an electric field in the semicon-

ductor, free electrons (in the conduction band) will be accelerated and thus gain energy. It is perfectly possible for such an electron to strike an electron in an energy level only slightly below the conduction band and excite it into conduction, without itself falling below the limit of the conduction band. Now there are two conduction electrons instead of one. This process is called "impact ionization: and is the source of "voltage effect" observed when photoresistors are subjected to electric field intensities of the order of 50 volts per inch of gap width or more. (Typical gap widths for photoresistor cells are of the order of .005 inches to .020 inches.) For the Clairex CL 605L-020 cell used in multipliers described in this thesis, this effect is of the order of -.075% resistance change per volt applied to the cell at moderate light levels and is slightly non-linear; the resistance change is proportional to the 1.17 power of the voltage.

From the foregoing, it is clear that both voltage effect and light history effect depend in detail on the occupancy of energy levels immediately below the conduction band. So far, we have tacitly assumed that all photons striking the cell had sufficient energy to excite electrons directly from the valence band to the conduction band. Now, suppose that a source providing substantial numbers of longer wave length photons is used. An incandescent lamp is such a source; an even better example is the common miniature neon lamp bulb. Such sources excite many electrons to energy levels immediately

below the conduction band thus effectively swamping the normal sources of electrons for these energy levels and making the trapping level population depend almost entirely on the instantaneous light level. Since the neon lamp has considerably more energy concentrated at wave lengths immediately longer than the wavelength required for excitation directly into the conduction band, this effect is very much more pronounced with these lamps. Thus, at a light level of 2 foot candles, a "time constant" of .045 seconds is typical for white light from an incandescent lamp. With illumination from a high brightness type miniature neon lamp of an intensity to give the same cell resistance, the "time constant" is typically .005 seconds. Notice however, that although the trapping levels are essentially optically filled on an increase of illumination, they still empty via the conduction band and thus the improvement of "time constant" for a decrease of illumination is not quite so great.

This improved response is not obtained without a price; since the levels immediately below the conduction band are much more heavily populated, the frequency of impact ionization is much greater and voltage effect is very much increased. For equal cell resistance, illumination with a high brightness neon lamp causes two to four times the voltage effect which occurs with illumination of an incandescent lamp. Use of a filter to remove longer wave length radiation from an incandescent lamp

reduces voltage effect by approximately 1/5.

FOOTNOTES

1. An excellent survey of the classical methods of analog multiplication is to be found in Korn and Korn, Electronic Analog and Hybrid Computers, Chapter Seven, "Electronic Multipliers and Dividers".

2. In a multiplier to be used in a hybrid computer other values may be useful. In multipliers of the feedback control type such as that to be discussed, $k = 1/Y_o$ (see Figure One) and it is possible to make k variable, i.e., to divide by Y_o. This idea appears in Korn & Korn and was also pointed out in a personal communication to the Electronic Systems Laboratory by Ulrich Tietze of Western Germany who has worked with photoconductive multipliers. The division scheme was not attempted with the multipliers described herein due to lack of time.

3. In the photoconductive multiplier, x is in the form of light which makes coupling to additional elements simple and eliminated "loading" difficulties. That is actually one of the chief advantages of multipliers of this type.

4. After Figure Two, page 4, of Connelly, An Analog Photoresistive Multiplier, Report ESL-FR-258. Other multiplier forms were suggested by Tietze in the communication previously mentioned.

5. Appendix IV contains a reasonably detailed physical explanation of these effects.

6. CdS data is from experimental work done in connection with this thesis;data on CdSe cells is adapted from the manufacturer's literature chiefly Clairex Photoconductive Cell Design Manual, 25CL366, 1966.

7. The manufacturer of the cells used suggested that the narrowest spread of temperature might be achieved by using cells from a single batch rather than cells selected for other characteristics from a number of batches; this idea was tested by measuring the temperature coefficients of two groups of cells with the following results:

Temperature Coefficient at 40,000 ohms at 28° C.

Single Batch - 15 cells		Mixed Batches - 8 cells
57 ohms/°C	73 ohms/°C	103 ohms/°C
63 "	76 "	144 "
64 "	76 "	129 "
64 "	78 "	110 "
65 "	85 "	93 "
66 "	87 "	89 "
69 "	94 "	78 "
71 "		47 "

Even by inspection one can get the idea that the single hatch cells had a more uniform temperature coefficient. This shows

7. cont.

up even more clearly when a few calculations are done. The mean temperature coefficient for the entire group of 23 cells is 81.8 ohms/^{o}C. The root mean square deviation from that mean is 33.05 ohms/^{o}C for the random group but only 13.56 ohms/^{o}C for the single-batch cells.

8. The diodes tested were Monsanto type MVE-100. These have efficiencies of the order of .1% or less and operate at a wavelength between 6000 and 7000 Å (visible red). Paul M. Hamilton, commercail Development Manager of the Semiconductor Materials Department stated in a personal communication to the writer dated 28 June 1966 that his firm was attempting to develop a diode in the 5500 - 5700 Å region (visible green). Such a device would be a much better match to the average CdS photoconductor which has peak sensitivity in the 5150 - 6200 Å region depending on doping. The CL 605L - 020 has peak sensitivity at about 5500 Å.

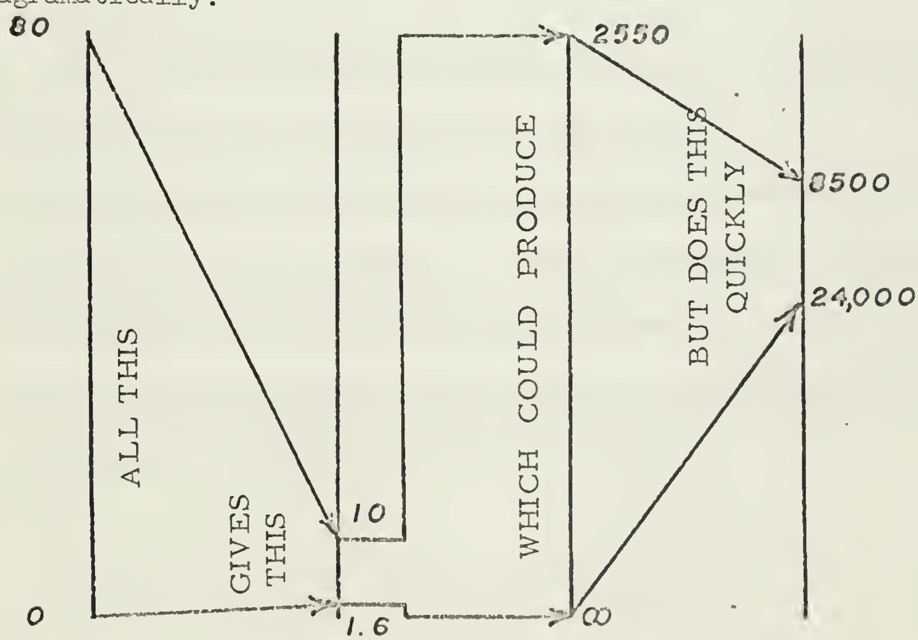
9. The limit would be the mechanical stresses from electric and magnetic fields or possibly thermal stress. None of these seems to be a problem, however.

10. The data on the Electronic Systems Laboratory four-quadrant multiplier are taken from Table I, page 6 of Report ESL-FR-258.

11. This is more of a problem than was anticipated. In the measurement of the temperature coefficients in Footnote #7, thermal gradients of the order of one degree Farenheight per degree per minute were observed despite multiple heating elements and careful insulation of the light cavity. Anderson in his thesis (An Analog Electro-Optical Multiplier, M.I.T., Department of Electrical Engineering, 1963) discusses this at some length.

12. A possible problem is variation in the distribution of light among the cells caused by movement of the lamp filament. Specific attention should be given to this question.

13. Diagramatically:



Range of dynamic lamp voltages | Range of static lamp voltages | Range of static cell resistances | Range of cell resistances for full scale

13. In other words, the static lamp intensity range is greater than that required for full scale output from the cells and the dynamic range of lamp voltages exceeds that needed for the full range of static lamp intensities.

14. A good reference is the "RCA Photocell Handbook" which describes the general characteristics of a variety of photosensitive devices.

15. In an early test, a lamp-cell-cavity combination operated on a square wave input from a relay at about 10 Hz. with a cell "time constant" of under 2 milliseconds for several hours without any lamp troubles. The peak lamp voltage was 25 volts as in the present case, but the range of static lamp voltages was different. The optimum relation should have been obtained at that time.

16. The trade off of stability for speed is forced by the physical principles involved as explained in Appendix IV.

17. The manufacturer's data provides only the gross outlines; the "voltage effect" for example, is not even mentioned. Semiconductor physics books, on the other hand, provide plenty of detail but don't make clear what principles cause which effects This appendix is an effort to bridge the gap.

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